

# PROBING THE EFFICIENCY OF ELECTRON-PROTON COUPLING IN RELATIVISTIC COLLISIONLESS SHOCKS THROUGH THE RADIO POLARIMETRY OF GAMMA-RAY BURST AFTERGLOWS

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## ABSTRACT

The late-time optical/radio afterglows of  $\gamma$ -ray bursts (GRBs) are believed to be synchrotron emission of electrons accelerated in relativistic collisionless shocks propagating in the ambient medium of the sources. However, the fraction  $f$  of electrons that are coupled to protons and accelerated remains unclear and a large number of thermal electrons that are not coupled to protons may be left behind. If  $f < 1$ , the true explosion energies of GRBs are  $f^{-1}$  times larger than those commonly estimated with  $f = 1$ . Thus the value of  $f$  gives an important constraint on the nature of the central engine of GRBs and the physics of collisionless shocks. Although early-time radio observations can probe the thermal electrons, they are difficult at present. We show that the Faraday rotation effects of the thermal electrons may suppress the linear polarization of the afterglow at frequencies higher than the absorption frequency in the late time, if the magnetic field is ordered at least in parts, and that  $f$  can be constrained through the observation of the effects. We find that those effects may be detected with late-time,  $\geq 1$  day, polarimetry with ALMA for a burst occurring within 1 Gpc (i.e.,  $z \simeq 0.2$ ), if  $f \sim 10^{-1}$ .

*Subject headings:* acceleration of particles — gamma rays: bursts — polarization — radio continuum: general

## 1. INTRODUCTION

The afterglows of  $\gamma$ -ray bursts (GRBs) have been observed mainly in the optical and radio wavebands in the late time (i.e., several hours after the burst trigger) since the late 1990's, and they are widely explained as due to synchrotron emission of electrons accelerated in relativistic collisionless shocks driven into the ambient medium of the GRB sources. The synchrotron emission mechanism is supported by the detection of linear polarization at the level of  $\sim 1-3\%$  in several optical afterglows (for reviews, Covino et al. 2004; Lazzati 2006). These understandings are also being confirmed by recent observations with *Swift* satellite, although the situation is very complicated in the X-ray band and in the early time (for reviews, Piran 2004; Mészáros 2006; Zhang 2007).

In the standard external shock model of GRB afterglows, the late-time dynamics of the shock is determined by the explosion energy  $E$  and the ambient medium number density  $n$ . We usually treat the fractions of the explosion energy that go into the magnetic field and electrons and the fraction of electrons that gain the energy of protons as free parameters  $\epsilon_B$ ,  $\epsilon_e$ , and  $f$ , respectively, since these have not been derived from the basic principles. We additionally assume that all the electrons that gain the proton energy are accelerated to form a power-law energy distribution,  $dn/d\gamma_e \propto \gamma_e^{-p}$  for  $\gamma_e \geq \gamma_m \simeq \epsilon_e(m_p/m_e)\Gamma$ , where  $\Gamma$  is the Lorentz factor of the shocked fluid, and a thermal component with  $\gamma_e \simeq \epsilon_e(m_p/m_e)\Gamma$  is not produced. We commonly constrain the parameters  $\{E, n, \epsilon_B, \epsilon_e, p\}$  by the observations, assuming  $f = 1$  (e.g., Panaitescu & Kumar 2002; Yost et al. 2003).

This implies that current observations cannot constrain the electron-proton coupling parameter  $f$ . Eichler & Waxman (2005) showed that the observations also allow the external shock model with the parameters chosen as  $\{E' = E/f, n' = n/f, \epsilon'_B = \epsilon_B f, \epsilon'_e = \epsilon_e f, p' = p\}$  in which the fraction  $f$  of

total electrons gains the proton energy and the fraction  $(1-f)$  is thermal electrons with  $\gamma_e \simeq \Gamma$ , as long as  $m_e/m_p < f < 1$ . (Hereafter we call the former and latter electron components “accelerated electrons” and “thermal electrons”, respectively.) The parameter  $f$  gives an important constraint on the true explosion energies of GRBs. It also gives a clue to unveil the physics of collisionless shocks.

The thermal electrons provide additional emission and absorption for the afterglow flux only for  $\nu < \tilde{\nu}_m$ , where  $\tilde{\nu}_m$  is the characteristic synchrotron frequency of the thermal electrons. This results in the sharp decline or sharp rise of flux as  $\tilde{\nu}_m$  passes the observed frequency, which can be detected through early-time radio observations,  $t \lesssim 10^3$  s for  $\nu \gtrsim 10^{11}$  Hz (Eichler & Waxman 2005). However, such observations are difficult at present. Furthermore, it is unclear whether the standard model is applicable for  $t \lesssim 10^3$  s (see e.g., Ioka et al. 2006; Toma et al. 2006).

In this paper, we show that the thermal electrons give the Faraday effects on the afterglow polarization substantially even in the late time and that  $f$  can be constrained through the observation of the effects. Those effects may be significant (even for  $\nu > \tilde{\nu}_m$ ) if the magnetic field is ordered to some extent (Matsumiya & Ioka 2003; Sagiv et al. 2004). The frequency below which the Faraday rotation effect is significant is expected to be orders of magnitude higher than that supposed so far with  $f = 1$ , because the position angle rotation depends on electron Lorentz factor as  $\Delta\chi \propto (\ln \gamma_e)/\gamma_e^2$ .

## 2. TRANSFER OF POLARIZED RADIATION

The transfer of polarized radiation through spatially homogeneous plasma with a weakly anisotropic dielectric tensor may be described by the transfer equation of Stokes parameters (Sazonov 1969; Jones & O’Dell 1977; Melrose 1980a,b),

$$\begin{pmatrix} d/ds + \kappa_I & \kappa_Q & 0 & \kappa_V \\ \kappa_Q & d/ds + \kappa_I & \kappa_V^* & 0 \\ 0 & -\kappa_V^* & d/ds + \kappa_I & \kappa_Q^* \\ \kappa_V & 0 & -\kappa_Q^* & d/ds + \kappa_I \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I \\ \eta_Q \\ 0 \\ \eta_V \end{pmatrix}, \quad (1)$$

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where  $s$  is a length parameter along the ray path, and the right-handed system of coordinates with the wavevector  $\mathbf{k}$  along axis 3 and the magnetic field  $\mathbf{B}$  on plane 2-3 is adopted. Here  $\eta_{l,Q,V}$  are polarization-dependent emissivity, and  $\kappa_{l,Q,V}$  ( $\kappa_{Q,V}^*$ ) are the transfer coefficients related to the anti-Hermitian (Hermitian) part of the dielectric tensor, describing polarization-dependent absorption (the polarization of the normal modes of the plasma). If  $|\kappa_V^*| \gg |\kappa_Q^*|$ , the normal modes are circularly polarized, and the transfer equation (1) indicates that the conversion of  $Q$  and  $U$  occurs. This is well-known Faraday rotation. If  $|\kappa_Q^*| \gg |\kappa_V^*|$ , the normal modes are linearly polarized and the conversion of  $U$  and  $V$  occurs. This is called Faraday conversion.

We define the optical depth  $\tau = \kappa_I s$ , the rotation depth  $\tau_V = |\kappa_V^*| s$ , and the conversion depth  $\tau_Q = |\kappa_Q^*| s$ . The properties of the solution of the transfer equation (1) are as follows. First, suppose that the absorption effect is not significant, i.e.,  $\tau \ll 1$ . In this case the equation (1) may be integrated easily (Melrose 1980b; Jones & O'Dell 1977). For  $|\kappa_V^*| \gg |\kappa_Q^*|$ , we obtain the linear polarization

$$\Pi_L = \frac{\sqrt{Q^2 + U^2}}{I} \simeq \frac{\eta_Q}{\eta_I} \left| \frac{\sin(\tau_V/2)}{\tau_V/2} \right|, \quad (2)$$

and the circular polarization is given by the intrinsic one,  $\Pi_C = |V|/I = |\eta_V/\eta_I|$ . For  $\tau_V \gg 1$ , the linear polarization damps. This results from the fact that the emission from different points through the source have its polarization plane rotated at different angles. Analogously, for  $|\kappa_Q^*| \gg |\kappa_V^*|$  and  $\tau_Q \gg 1$ , the damping of  $\Pi_C$  occurs while  $\Pi_L$  remains intrinsic.

Secondly, in the case in which the absorption effect is significant, i.e.,  $\tau \gg 1$ , we may obtain the polarization degrees approximately by eliminating the differential term from equation (1). As an example, if the Faraday effects are weaker than the absorption effect, i.e.,  $\kappa_I^2 \gg \kappa_V^2$  and  $\kappa_I^2 \gg \kappa_Q^2$ ,

$$\Pi_L \simeq \left| \frac{\eta_Q/\eta_I - \kappa_Q/\kappa_I}{1 - (\eta_Q/\eta_I)(\kappa_Q/\kappa_I)} \right| \quad (3)$$

is obtained to the leading order. The circular polarization is similarly given by  $\simeq |\eta_V/\eta_I - \kappa_V/\kappa_I|$ .

In the following sections, we apply this formulation to the late-time GRB afterglows. The anisotropic part of the dielectric tensor is tens of magnitude smaller than unity for the shocked plasma of a typical GRB afterglow. We assume that (1) the pitch-angle distribution of electrons is isotropic for simplicity; (2) the shocked plasma is spatially homogeneous; (3) the shocked plasma consists of a number of random cells within each of which magnetic field is ordered. With the third assumption, we obtain the observed linear and circular polarizations by  $1/\sqrt{N}$  times those for completely ordered magnetic field, where  $N$  is the number of the random cells in the visible region (Jones & O'Dell 1977; Gruzinov & Waxman 1999). To reproduce the optical detection at the level of  $\sim 1-3\%$  (Covino et al. 2004),  $N$  would be  $\sim 10^3$ .

### 3. POLARIZATION OF LATE-TIME GRB AFTERGLOWS

In this section, we derive the polarization spectrum of the late-time afterglow, based on the standard external shock model in which all the electrons are accelerated, i.e.,  $f = 1$  (see § 1). The energy distribution of the electrons is assumed to be  $dn/d\gamma_e \propto \gamma_e^{-p}$  for  $\gamma_e \geq \gamma_m$ . The transfer coefficients

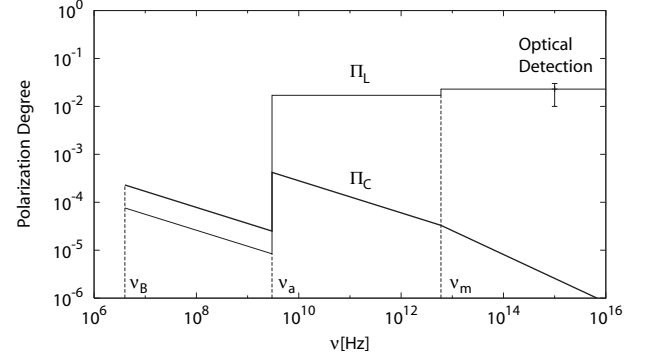


FIG. 1.— Polarization spectrum of a late-time GRB afterglow, at  $t = 1$  day, inferred from the standard external shock model in which all the electrons are accelerated, i.e.,  $f = 1$ . The degrees of linear polarization  $\Pi_L$  (thin solid line) and circular polarization  $\Pi_C$  (thick solid line) are shown. The degrees are calculated as  $1/\sqrt{N} \sim 10^{-1.5}$  times those for completely ordered magnetic field, i.e., they are calibrated by detected optical linear polarization (Covino et al. 2004). Typical values of parameters are used:  $E = 10^{52}$  ergs,  $n = 1 \text{ cm}^{-3}$ ,  $\epsilon_B = 10^{-2}$ ,  $\epsilon_e = 10^{-1}$ , and  $p = 2.2$ .

for such electron plasma are summarized for frequency region  $\nu > \nu_m$  by Jones & O'Dell (1977) and for  $\nu_B \ll \nu \ll \nu_m$  by Matsumiya & Ioka (2003),<sup>4</sup> where  $\nu_m$  is the characteristic synchrotron frequency corresponding to  $\gamma_m$  and  $\nu_B$  is the nonrelativistic electron Larmor frequency.

The radius of the shock and the Lorentz factor of the shocked fluid evolve as  $R \simeq (17Et/4\pi m_p c n)^{1/4}$  and  $\Gamma \simeq (17E/1024\pi m_p c^5 n t^3)^{1/8}$ , respectively, where  $t$  is the observer time (Sari et al. 1998). The comoving width of the shocked plasma shell can be estimated by  $R/4\Gamma$ , which we use as the path length of the transfer equation (1). The magnetic field strength, the minimum Lorentz factor and number density of the accelerated electrons are written by  $B = (32\pi m_p c^2 \epsilon_B n)^{1/2} \Gamma$ ,  $\gamma_m = \epsilon_e (m_p/m_e) \Gamma$ , and  $n_{\text{acc}} = 4\Gamma n$ , respectively. Then we obtain  $\nu_B \simeq 4 \times 10^6 E_{52}^{1/4} n_0^{1/4} \epsilon_{B,-2}^{1/2} \epsilon_{e,-1}^{-3/4} \text{ Hz}$  and  $\nu_m \simeq 6 \times 10^{12} E_{52}^{1/2} \epsilon_{B,-2}^{1/2} \epsilon_{e,-1}^2 t_d^{-3/2} \text{ Hz}$ , respectively. Here (and hereafter) we have adopted the notation  $Q_x = Q/10^x$  in cgs units and  $t_d = t/1$  day.

Figure 1 illustrates the polarization spectrum of the late-time GRB afterglow. The frequencies at which  $\tau$ ,  $\tau_V$ , and  $\tau_Q$  equal unity are given by  $\nu_a \simeq 3 \times 10^9 E_{52}^{1/5} n_0^{3/5} \epsilon_{B,-2}^{1/5} \epsilon_{e,-1}^{-1} \text{ Hz}$ ,  $\nu_V \simeq 10^9 E_{52}^{3/16} n_0^{9/16} \epsilon_{B,-2}^{1/4} \epsilon_{e,-1}^{-1/16} \text{ Hz}$ , and  $\nu_Q \simeq 10^9 E_{52}^{1/5} n_0^{3/5} \epsilon_{B,-2}^{1/5} \epsilon_{e,-1}^{-1} \text{ Hz}$ , where  $p = 2.2$  has been used as a fiducial value. Since  $\nu_a > \nu_Q \simeq \nu_V$ , so that no plasma effects are significant in the optically thin regime  $\nu > \nu_a$  and the intrinsic degree of polarization is obtained,  $\Pi_L = \eta_Q/\eta_I = 0.5$  and  $\Pi_C = |\eta_V/\eta_I| \simeq \gamma_m^{-1}(\nu/\nu_m)^{-1/3}$  for  $\nu \ll \nu_m$ . For  $\nu > \nu_m$ ,  $\Pi_L = (p+1)/(p+7/3) \simeq 0.7$  and  $\Pi_C \simeq \gamma_m^{-1}(\nu/\nu_m)^{-1/2}$ . In the optically thick regime  $\nu < \nu_a$ ,  $\tau^2 \gg \tau_V^2$  and  $\tau^2 \gg \tau_Q^2$  are satisfied, the linear polarization is given by equation (3). Because  $\eta_Q/\eta_I = \kappa_Q/\kappa_I = 0.5$  for  $\nu \ll \nu_m$ , the intrinsic linear polarization vanishes and  $\Pi_L$  is only produced by the conversion of the circular polarization. The transfer equation (1) indicates that  $\Pi_L \approx (\kappa_Q^*/\kappa_I)(\eta_V/\eta_I - \kappa_V/\kappa_I) \approx 2 \times 10^{-2} |\eta_V/\eta_I|$  and  $\Pi_C \approx \eta_V/\eta_I - \kappa_V/\kappa_I \approx 6 \times 10^{-2} |\eta_V/\eta_I|$ . All the characteristic frequencies  $\nu_a$ ,  $\nu_Q$ , and  $\nu_V$  are weakly dependent of time, so that the polarization spectrum does not evolve significantly.

<sup>3</sup> Electron cooling makes the electron energy distribution inhomogeneous, but it can be neglected in the late phase of the afterglow (Sari et al. 1998).

<sup>4</sup> We adopt the value of  $\kappa_{\alpha}^* = (\alpha + \frac{3}{2})/2$  different from that shown in Jones & O'Dell (1977), and the sign of  $\kappa_V$  should be changed in Matsumiya & Ioka (2003).

The suppression of  $\Pi_C$  due to absorption effect has not been pointed out in the context of GRB afterglows, since Matsumiya & Ioka (2003) erroneously used  $\kappa_V$  of opposite sign and Sagiv et al. (2004) neglected  $\kappa_V$ . Sagiv et al. (2004) discussed similar propagation effects in early-time afterglows, but their discussion should be restricted to a frequency region  $\nu \gg \nu_B (\simeq 10^{10} - 10^{11} \text{ Hz})$ . For  $\nu \gg \nu_B (> \nu_a)$ ,  $\Pi_L$  and  $\Pi_C$  they derived for the forward shock emissions are consistent with our results.

#### 4. A SIGNATURE OF THE THERMAL ELECTRONS

Here we derive the polarization spectrum according to the standard external shock model in which the thermal electrons are left behind, i.e.,  $f < 1$  (see § 1), and show that the linear polarization may be suppressed even at frequencies higher than the absorption frequency  $\nu_a$ . The electron energy distribution is assumed to consist of the accelerated electrons which are considered in § 3 and the thermal electrons with the Lorentz factor  $\tilde{\gamma}_m = \Gamma$  and the number density  $n_{th} = [(1-f)/f]n_{acc}$ , where  $m_e/m_p < f < 1$  (see Fig. 1 of Eichler & Waxman 2005). Then all the quantities in this model can be written by using the parameters  $\{E, n, \epsilon_B, \epsilon_e, p\}$  as measured assuming  $f = 1$ , while the real values of the parameters are given by  $\{E' = E/f, n' = n/f, \epsilon'_B = \epsilon_B f, \epsilon'_e = \epsilon_e f, p' = p\}$ . The characteristic synchrotron frequency of the thermal electrons is estimated by  $\tilde{\nu}_m \simeq 2 \times 10^8 E_{52}^{1/2} \epsilon_{B,-2}^{1/2} t_d^{-3/2} \text{ Hz}$ . We approximate the transfer coefficients for the thermal electrons as those for the monoenergetic distribution of electrons (Sazonov 1969; Melrose 1980a,b). Thus we consider the electron energy distribution

$$\frac{dn}{d\gamma_e} = n_{th} \delta(\gamma_e - \tilde{\gamma}_m) + K \gamma_e^{-p} H(\gamma_e - \tilde{\gamma}_m), \quad (4)$$

where  $K = (p-1)n_{acc}\gamma_m^{p-1}$  and  $H(x)$  is the Heaviside step function. (Hereafter we describe the quantities related to the thermal electrons as  $\tilde{Q}$ .) For  $\nu \gg \tilde{\nu}_m$ ,  $\tilde{\eta}_{L,Q,V}$  and  $\tilde{\kappa}_{L,Q,V}$  damp exponentially. The remaining coefficients for the thermal electrons are different from those for the power-law distribution only by numerical factors. Here we show the expressions of Faraday coefficients,

$$\begin{aligned} \tilde{\kappa}_V^* &\simeq \frac{1}{\pi} \frac{e^2}{m_e c} n_{th} (2\pi\nu_B \cos\theta) \tilde{\gamma}_m^{-2} (\ln \tilde{\gamma}_m) \nu^{-2}, \\ \tilde{\kappa}_Q^* &\simeq \begin{cases} \frac{2^{1/3} \pi^{1/3}}{3^{11/6} \Gamma_E(\frac{1}{2})} \frac{e^2}{m_e c} n_{th} (2\pi\nu_B \sin\theta)^{2/3} \tilde{\gamma}_m^{-5/3} \nu^{-5/3} & \text{for } \nu \ll \tilde{\nu}_m, \\ -\frac{1}{2\pi^2} \frac{e^2}{m_e c} n_{th} (2\pi\nu_B \sin\theta)^2 \tilde{\gamma}_m \nu^{-3} & \text{for } \nu \gg \tilde{\nu}_m, \end{cases} \end{aligned} \quad (6)$$

where  $\theta$  is the angle between  $\mathbf{k}$  and  $\mathbf{B}$  and  $\Gamma_E(x)$  is the Euler gamma function. The coefficients for the electron energy distribution consisting of the thermal plus accelerated ones are given by the linear combination of the two contributions.

In Figure 2, we show the linear polarization spectrum of the late-time GRB afterglow for the  $f < 1$  model, compared with that for the  $f = 1$  model obtained in § 3. The sensitivities of ALMA and VLA for 1 hour integration time are also shown. They are derived by  $\Pi_L \geq F_{s,\nu}/F_\nu$ , where  $F_{s,\nu}$  is the sensitivity limit for continuum radiation and  $F_\nu$  is the flux of the afterglow. The flux is estimated by  $F_\nu = F_m(\nu/\nu_m)^{1/3}$  for  $\nu_a < \nu < \nu_m$  and  $F_\nu = F_m(\nu_a/\nu_m)^{1/3}(\nu/\nu_a)^2$  for  $\tilde{\nu}_m < \nu < \nu_a$ , where  $F_m \simeq 10^2 D_{27.5}^2 E_{52} n_0^{1/2} \epsilon_{B,-2}^{1/2} \text{ mJy}$  and  $D$  is the luminosity distance (Sari et al. 1998).

For  $\nu \gg \tilde{\nu}_m$ , the absorption effect of the thermal electrons is absent, and thus the absorption frequency is

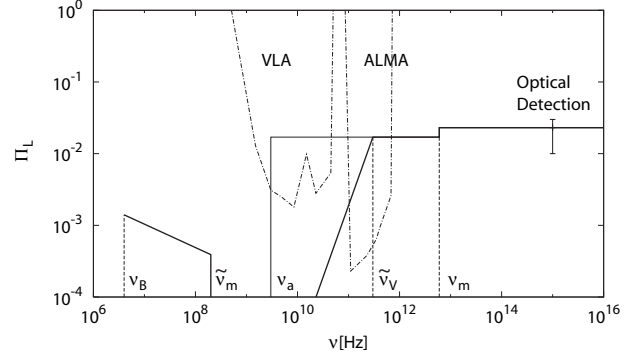


FIG. 2.— Linear polarization spectra of a late-time GRB afterglow, at  $t = 1$  day, inferred from the standard external shock models in which all the electrons are accelerated (thin solid line; the same of  $\Pi_L$  in Figure 1) and in which the thermal electrons are left behind with  $(1-f)/f = 10$  (thick solid line). The parameters  $E, n, \epsilon_B, \epsilon_e, p$ , and  $N$  are the same as Figure 1. The dot-dashed lines describe the sensitivities of VLA and ALMA for a burst with  $D = 1$  Gpc ( $z \simeq 0.2$ ) and an integration time of 1 hour. At frequencies lower than  $\tilde{\nu}_V$ ,  $\Pi_L$  is suppressed by the Faraday rotation effect of the thermal electrons, which can be detected with ALMA.

the same as the  $f = 1$  case. Since  $\tilde{\tau}_V/\tau_V \simeq [(1-f)/f](\tilde{\gamma}_m/\gamma_m)^{-2}(\ln \tilde{\gamma}_m/\ln \gamma_m) \gg 1$  (see equation (5)) and similarly  $\tilde{\tau}_Q/\tau_Q \gg 1$ , the Faraday effects are dominated by those of the thermal electrons. The ratio  $\tilde{\tau}_V/\tilde{\tau}_Q \simeq \tilde{\gamma}_m^{-1}(\ln \tilde{\gamma}_m)(\nu/\tilde{\nu}_m)$  is  $\gg 1$  for small  $\tilde{\gamma}_m$ , i.e., at the late phase of the afterglow, so that the normal modes of this plasma is circularly polarized and the Faraday rotation effect is significant. The frequencies at which  $\tau_V$  and  $\tau_Q$  equal unity are given by

$$\tilde{\nu}_V \simeq 3 \times 10^{11} \left[ \frac{(1-f)/f}{10} \right]^{1/2} E_{52}^{3/16} n_0^{9/16} \epsilon_{B,-2}^{1/4} t_d^{-1/16} \text{ Hz}, \quad (7)$$

and  $\tilde{\nu}_Q \simeq 4 \times 10^{10} [(1-f)/10f]^{1/3} E_{52}^{1/3} n_0^{1/3} \epsilon_{B,-2}^{1/3} t_d^{-2/3} \text{ Hz}$ , respectively. For  $\nu > \tilde{\nu}_V$ , all the depths are smaller than unity, so that the intrinsic polarization is obtained. In the regime  $\nu_a < \nu < \tilde{\nu}_V$ ,  $\tilde{\tau}_V \gg \tilde{\tau}_Q \gg 1 \gg \tau$  is satisfied, so that  $\Pi_L$  is given by equation (2). It damps at low frequencies as  $\propto \nu^2$  and oscillates with the period  $|\Delta\nu/\nu| \sim 10^{-1} \nu_{11}^2$ . In the optically thick regime  $\tilde{\nu}_m \ll \nu < \nu_a$ , the transfer equation (1) for  $\tilde{\tau}_V \gg \tilde{\tau}_Q \gg \tau \gg 1$  indicates that  $\Pi_L \approx (\tilde{\tau}_Q/\tilde{\tau}_V)(\eta_V/\eta_L - \kappa_V/\kappa_L)$  and  $\Pi_C \approx \eta_V/\eta_L - \kappa_V/\kappa_L$  (Jones & O'Dell 1977), and thus  $\Pi_L$  does not exceed  $\Pi_C$ . For  $\nu \ll \tilde{\nu}_m$ , both the absorption and the Faraday effects are dominated by the thermal electrons, and  $\tilde{\tau}^2 \gg \tilde{\tau}_V^2$  and  $\tilde{\tau}^2 \gg \tilde{\tau}_Q^2$  are satisfied. Then the polarization spectrum is similar to that for  $\nu < \nu_a$  in the  $f = 1$  model discussed in § 3. It is important to note that both  $\Pi_L$  and  $\Pi_C$  are  $< 10^{-2}$  for  $\nu < \nu_a$ , and they are far from detectable because the flux is suppressed in this regime (especially for  $\nu < \tilde{\nu}_m$ , the additional absorption by the thermal electrons exists).

The existence of the thermal electrons is characterized by the suppression of the linear polarization at  $\nu_a < \nu < \tilde{\nu}_V$ . Necessary conditions for this suppression are  $\tilde{\nu}_V \gg \nu_a$  and  $\tilde{\nu}_V \gg \tilde{\nu}_Q$ . The former condition reduces to  $(1-f)/f \gg 10^{-3} E_{52}^{1/40} n_0^{3/40} \epsilon_{B,-2}^{-1/10} \epsilon_{e,-1}^{-2} t_d^{1/8}$ . Interestingly, the effect can be seen even for as small number of thermal electrons as  $(1-f)/f \sim 10^{-1}$ . The latter condition implies that  $t \gg 3 \times 10^3 [(1-f)/f]_1^{-8/29} E_{52}^{7/29} n_0^{-11/29} \epsilon_{B,-2}^{4/29} \text{ s}$ , which shows that the late-time afterglow is suitable to search for the existence of the thermal electrons through the observation of linear polarization.

If  $\nu_a$  is determined by the observation of a bright burst and the linear polarization is not detected at  $\nu \gtrsim \nu_a$  with VLA and detected at  $\nu \gg \nu_a$  with ALMA, it becomes clear that a

number of the thermal electrons exist and the magnetic field is ordered on large scales. If we determine  $\tilde{\nu}_V$ , the electron-proton coupling parameter  $f$  can be constrained by equation (7).

### 5. DISCUSSION

We have studied a signature of the thermal electrons only in the patchy coherent magnetic field model, while there are some other viable models for magnetic field configuration. In the model of random field with very short coherence length (e.g., Sari 1999; Ghisellini & Lazzati 1999), the coefficient  $\tilde{\kappa}_V^*$  averaged over the field configuration vanishes, so that the Faraday depolarization of  $\Pi_L$  does not occur (Matsumiya & Ioka 2003). In the model of a combination of random field  $B_{\text{rnd}}$  and large-scale ordered field  $B_{\text{ord}}$  (Granot & Königl 2003), the depolarization by  $B_{\text{ord}}$  can occur similarly as discussed in § 4. In this model  $\Pi_L \simeq 0.7B_{\text{ord}}^{1.6}/\langle B_{\text{rnd}}^{1.6} \rangle$  for  $\nu > \nu_m$  and  $p = 2.2$ , so that  $B_{\text{ord}}^{1.6}/\langle B_{\text{rnd}}^{1.6} \rangle \sim 10^{-1.5}$  to reproduce the optical detection. Interestingly for  $\tilde{\nu}_V < \nu \ll \nu_m$ ,  $\Pi_L = 0.5B_{\text{ord}}^{2/3}/\langle B_{\text{rnd}}^{2/3} \rangle \sim 0.1$ . If such a high  $\Pi_L$  at  $\tilde{\nu}_V < \nu \ll \nu_m$  is detected, it will be an evidence for the presence of  $B_{\text{ord}}$ .

Only upper limits have been obtained so far for the radio polarization from GRB afterglows (Granot & Taylor 2005). From bright GRB 030329,  $\Pi_L \simeq 2\%$  is measured in the optical band (Greiner et al. 2003), whereas  $3\sigma$  limits  $< 1\%$  are derived at 8.4 GHz. Such a low degree at radio may be attributed to the source being optically thick, since  $\nu_a$  is estimated as  $\simeq 19$  GHz (Taylor et al. 2005).

If a large number of thermal electrons are left behind, i.e.,  $f < 1$ , the afterglow energy of GRBs should be  $E' = Ef^{-1}$ , where  $E$  is the afterglow energy estimated by  $f = 1$  model and typically inferred to be  $\sim 10^{51.5}$  ergs with jet collimation correction (Panaitescu & Kumar 2002; Yost et al. 2003). The

association of GRBs with supernovae suggests that  $f > 10^{-1.5}$  is a conservative lower limit. The energy of prompt  $\gamma$ -ray emission is typically similar to  $E$ , and some of the models of early-time afterglows imply that the efficiency of the  $\gamma$ -ray emission is  $\gtrsim 90\%$  (e.g., Ioka et al. 2006; Toma et al. 2006; Granot et al. 2006; Fan & Piran 2006). If the external shock model with  $f < 1$  is applicable to early-time afterglows, the  $\gamma$ -ray efficiency problem would be solved.

Mundell et al. (2007) have reported a  $2\sigma$  upper limit  $< 8\%$  on the optical polarization in early-time afterglow of GRB 060418 ( $t \sim 200$  s), and argued that presence of a large-scale ordered field in the GRB jet is ruled out. However, the rotation frequency  $\nu_V$  for a typical reverse shocked ejecta with ordered field is  $\sim 10^{15}$  Hz, so that the low level of polarization degree would result from the Faraday depolarization (see Sagiv et al. 2004).

It is suggested that there are electrons well coupled to protons at some nonrelativistic collisionless shocks, that is,  $\epsilon_e \approx 0.5$  (e.g., Markevitch 2006). However, the fraction  $f$  of total electrons that are coupled to protons has not been discussed seriously. Recently, Spitkovsky (2007) has reported the results of 2-dimensional particle-in-cell simulations of relativistic collisionless shocks in electron-proton plasma with realistic value of  $m_p/m_e$  that  $\epsilon_e \approx 0.5$  and  $f \approx 1$  were realized. However, it is too early to interpret the results conclusive, since long-term 3-dimensional simulations with good resolution have not been done and it has not been understood whether the heated electrons are accelerated into the power-law energy spectrum. The late-time radio polarimetry may be an important test for more realistic simulations and theories.

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